BRIDGING MATH AND MUSIC

by

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ABSTRACT

BRIDGING MATH AND MUSIC

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The goal of this project is multifaceted. The surface goal is to explore and explain the nature of the relationship between math and music. This tip-of-the-iceberg concept further branches into two interrelated goals: to revive interest in interdisciplinary studies, and to function as a treatise on the question of "what is beauty?" The cumulative result of these goals has manifested in a musical composition that combines math and music, bridging the two fields and functioning as a paradoxical work of art that is based almost entirely in math. This paradox comes, in-part, from a desire to avoid a strict sense of Rameau's concept of tonality. While parts of this work are in the sonic envelope of tonal music, others delve into tone-clusters, retrograde inversion at the tritone, and uncommon rhythmic sequences that lend themselves more to the ideologies of 20th century modernists. This fugue, after all, is an homage to music old and new: old in which math and music were of the same discipline, and new with the violent reproach post-Schoenbergian composers (sometimes) approached tonality with.

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CHAPTER ONE: INTRODUCTION

From the time of Classical Greece, and perhaps further still, music and math have had a symbiotic relationship. Lore has it that Western tonality was derived when Pythagoras witnessed a blacksmith hammering at his craft. Pythagoras heard the relations of the different pitches as the hammer hit and upon closer inspection noted the ratios of where the hammer hit in relation to the different pitches (Fauvel 14). This initial application and relationship between math and music created the scale that Western music still uses to this day—albeit with a slightly different tuning system. Since the creation of this scale, math and music have further developed together in rhythm and meter, systems of tuning, and even the sonic spectrum in which we aurally perceive different timbres. Recently, apparent emphases toward specificity within disciplines have limited important interdisciplinary scholarship. This thesis is an attempt to rekindle the relationship between two of the Quadrivium from ancient times—math and music.

My thesis delves into the symbiotic relationship between these two fields. These relationships deal with everything from pitch (production, tuning, and timbral quality) and rhythm (quasi-integral serialism) to scales used (the Fibonacci and overtone scale) and even to the form of the piece as a whole (use of the Golden Ratio). Harkening back to the days of Pythagoras, music was not considered an art—it was deemed a science (Grout 39). While I believe music is more art than science, I appreciate the Greeks presenting the primary understanding of harmonic relations, and the mathematical approach they initially took with music.

CHAPTER TWO: ANALYSIS

My approach to bridging the gap between math and music begins with the composition of a new piece. This piece pays homage to many of the eras of music history, while also consciously incorporating mathematical premises. The manifestation of this piece is a fugue, one of the most prominent musical genres of the so-called "classical" era[1], yet with a 20th century approach to tonality. Having written works in both of these styles before (a fugue in 18th century counterpoint and a 12-tone piece based completely around symmetry and reflection), this project has merged the two together. Preparation for writing this work included analysis of some of the most important collections of fugues composed throughout the ages. These include Bach's *Well-Tempered Clavier*, Shostakovich's *Preludes and Fugues*, and Hindemith's *Ludus Tonalis*.

My piece represents a return to ancient music in many respects, yet with a new, post-modern approach. This methodology is interdisciplinary in nature, applying math to music composition, but not simply for the novelty of it. This is also a treatise on the beauty in math, seen consistently in nature, and now to be made incarnate in a composition.

The piece has four different musical underpinnings (further referred to as rows, ex. P_F, P_H, etc.), each of sixteen notes: use of the Fibonacci scale for the subject, the first sixteen notes of the harmonic series arranged in such a manner as to function as a countersubject, a major scale, and the circle of fifths repeated twice through. These four rows already have somewhat of a traditional harmonic relationship amongst them, yet I hope the fugue has functioned as a catalyst in further glorifying these hidden relations. Some of the mathematical techniques I have applied in composition include basic spectral theory (i.e. use of the harmonic series), the golden ratio (in regards to form), and semi-serialized use of rhythm and meter.

[1] That era spanning the Baroque to the Romantic

X: Rhythm

The function of rhythm throughout the majority of the piece alludes to the Fibonacci series. This series (1, 1, 2, 3, 5, etc…) is most easily observed in the 2+3+5:4 time signature. While some would immediately call attention to the missing '1's, it should be noted that the eighth note anacrusis at the beginning of the piece (when coupled with the preceding eighth rest) functions as the absent '1's.

Rhythm is also developed within the first middle entry of the piece. The entry itself is a canon with both pitch and rhythmic relations based off of the primary row in the bass line, presented in the tuba. From there pitch content and temporal misalignment are proportionally developed in all parts but the tenor (trombone) part. The function of this row has an inversely proportional relationship. So as the rhythm the trombone plays is 2/3 the temporal value of the notes the tuba is playing, the pitch ratio is 3/2 (ex. A440*3/2 = 440*1.5 = 660 = E660). This presents a temporally misaligned dilemma as the trombone is literally playing against the temporal envelope of the implied 5:4 time signature as well as the perfectly related bass, alto, and soprano lines.

A final rhythmic nuance deals with the retrograde rhythms that occur in the subject and countersubject in the second middle entry. These can be viewed as reflections across the x-axis (i.e. the unit and element of *time* in a piece of music) of a Cartesian plane.

Y: Sound

The use of sound in this piece is derived almost entirely from 4 different rows[2]:

P_F (Fibonacci)	C C D E G C A B A A G F D C A B C
P_H (Harmonic Series)	C C G C E G B-flat C D E F-sharp G A B-flat B-natural C
P_I (Ionian/Major scale)	C D E F G A B C C B A G F E D C
and P_C (Circle of 5^{ths})	C F B E A D G C (C F B E A D G C)

[2] In this example all of the rows will begin on a 'C.'

Use of the Fibonacci scale as a row is perhaps most unfamiliar to disciples of both math and music alike. The scale is derived from use of the Fibonacci series (i.e. 1, 1, 2, 3, 5, 8, etc.) and lining it up against the seven notes of Western music (not taking into account the chromatic notes). Interestingly enough, the cycle repeats itself every 16 notes. While this could have been applied to any mode, I applied it to a C-based "Ionian" mode.

Thus:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17 (repeat)
1	1	2	3	5	8 (octave, simplify to 1)	13 (simplify to 6)	21 (7)	34 (6)	55 (6)	89 (5)	144 (4)	233 (2)	377 (1)	610 (6)	987 (7)	1597 (1)
C	C	D	E	G	C	A	B	A	A	G	F	D	C	A	B	C

Jason Stokes has the proof for this "scale" on his website[3]:

> To prove the Fibonacci sequence under modulo seven has a finite order of sixteen, we can use mathematical induction…
>
> Since it's known that $F_{n+1} = F_n + F_{n-1}$, let's assume that:
>
> $$Mod\ (Fn+1, 7) = Mod\ (Fn + Fn-1, 7)$$
>
> Or in other words:
>
> $$Mod\ (Fn+1, 7) = Mod\ (Mod\ (Fn, 7) + Mod\ (Fn-1, 7), 7)$$
>
> This holds true for our base cases, n = 1 and n = 2.
>
> For our inductive step (to prove the sequence repeats every sixteenth natural number) let's assume that:
>
> $$Mod\ (F(n+1)+16, 7) = Mod\ (F(n+16) + F(n-1)+16\ ,7)$$
>
> *OR*
>
> $$Mod\ (F\ n+17, 7) = Mod\ (Mod\ (Fn+16, 7) + Mod\ (F\ n+15, 7), 7)$$

[3] http://jonathanwstokes.com/2011/02/20/fibonacci-scale/

OR

$$Mod\ (2584, 7) = Mod\ (Mod\ (1597, 7) + Mod\ (987, 7), 7)$$

OR

$$1 = 1$$

AND

$$Mod\ (F_n, 7) = Mod\ (F_{n+16}, 7)$$

I came across this scale in my own pursuit of the relation between math and music. My "proof" dealt with finding a recurring pattern in the remainder when each number of the Fibonacci sequence was divided by 7 (again, the number of named notes in Western music). I was anticipating creating a very different kind of piece with the resulting notes, yet was surprised when I realized that the 16 notes initially found repeated indefinitely.

This collection of 16 notes led me to investigate the relationship between the Fibonacci scale (P_F) and the harmonic series (P_H). The harmonic series (in math) is most easily defined as 1/n. In relation to music this series is also known as the overtone series, and is what gives sound unique timbral properties. The collection of notes associated with the overtone series is given when multiplying the fundamental frequency by ascending integers (thus A440*2=A880, A440*3=E1360, A440*4=A1760, etc.). Brass instruments are perhaps the most closely related to the harmonic series as every "open" note (that is, without any depressed valves) is derived from frequency ratios of the fundamental note (B-flat for most brass instruments).

1	2	3	4	5	6	7	8
58.271 Hz	116.541 Hz	174.812 Hz	233.082 Hz	291.353 Hz	349.623 Hz	407.894 Hz	466.164 Hz
B-flat	B-flat	F	B-flat	D	F	A-flat	B-flat
Fundamental	Octave	Fifth	Octave	Third	Fifth	Flat Seventh	Octave

The other two rows I selected (a major scale, and the Circle of 5^{ths}) characterize tonal music. The relation between the two typifies Rameau's treatise on tonality (as opposed to modality) and harmony and set the precedent for such composers as Johann Sebastian Bach, Wolfgang Amadeus Mozart, and Ludwig van Beethoven. Of particular interest to this thesis are the works of Bach, the composer who wrote three definitive collections of fugues.

Most interesting to me was the quasi-tonal relationship the four rows had when juxtaposed against one another. With very minute finagling it was possible to create a harmonic progression that descended into utter chaos, only to resolve in a V-I perfect authentic cadence. This progression is manifested right at the climax of the piece, only to immediately descend into the aforementioned retrograde middle entry.

Z: Form and Other Considerations

Use of the golden ratio in regards to form is an appropriation from nature, the visual arts, and even architecture of which it has long been considered visually appealing (The Golden Ratio). This ratio is usually expressed as A:B being the same ratio as (A+B):C, but can also be understood as being approximately 61.8% of a whole. In the earlier 12-tone piece that I wrote, one of the few changes I made across the mirror reflection of the piece was a complete silence at the golden section. Silence, after all, is golden. Dr. Clement Falbo, however, has argued that society's mass labeling of the golden section throughout disciplines is incorrect (Falbo). Be this as it may, this particular composition was very consciously created with special attention to the golden section.

The climax of this piece occurs at the golden section with a displaced 1:4 measure (m.12 is "missing" a beat) beat of silence at the positive golden section (m.38). The first time all five voices are heard is at the negative golden section (m.18). Use of the golden section is also

present in the exposition of the fugue. At the negative golden section the first trumpet has two consecutive large leaps (m.5), and at the positive golden section of the exposition there is a hidden tone cluster disguised by displaced octaves, but brought to attention by a *sforzando* in the second trumpet part (m.8).

Though the piece is called a fugue, it should be noted that it does not abide by many of the conventions of an 18^{th} century fugue nor 18^{th} century counterpoint. The first discrepancy can be found in the absence of any true sense of tonality. Tonality is indispensable in this era's counterpoint, with the different species all reliant upon varying degrees of consonance and dissonance. Aside from the free use of dissonance throughout the piece, modulations to relative keys do not occur—per say. "Modulations" instead translate to rows based on harmonic series with different fundamentals. The function of the harmonic series at n=7 suddenly changes from a collection of pitches outlining a tonic triad to that of a dominant chord, more specifically a secondary dominant of an implied subdominant. Use of this in the first episode allowed for a "modulation" from a B-flat based harmonic series to an E-flat-centric series.

A secondary satirization of the fugue is further non-traditional use of episodes. These sections typically use fragments of the subject or countersubject to modulate to different keys, generally through the use of sequences. This is entirely abandoned in the episodes of this composition. The first episode relies on the pitch collection of the harmonic series (P_H) and prefaces the ratio canon with contrasting temporal envelopes. The second episode uses the four rows juxtaposed against each other, and was the first material composed for the piece. It is written in the style of a chorale, and has no aurally perceivable melodic precedent, yet leads to the climax (and golden section) of the piece. The third and final episode is similar to the second as it is in somewhat of a chorale voicing, yet it is based only on the circle of fifths.

The final entry borrows and combines elements from the rest of the work. The horn, the only instrument to not yet begin the subject, starts a solo recounting of this main theme of the work. The tuba continues the subject as the horn goes into the counter subject, yet is immediately interrupted by the jarring rhythms of the trombone playing the subject as it did in the ratio canon. The trumpets enter together in the next measure, showing a more cohesive duo as the first trumpet plays the subject and the second trumpet simultaneously plays the counter subject. By this point both the horn and the tuba have abandoned their former roles and are relying on two of the rows to finish the phrase: the horn playing a chromatic scale (an extension of P_H where n>16), and the tuba cycling through the circle-of-fifths one final time. The piece ends with a plagal cadence, a final homage to the Baroque era and J.S. Bach.

As previously mentioned, this work's purpose is multifaceted. It is an homage to music of both the past and the present, taking the form or "style" (Kenan, 201) of the 18th century fugue and adapting it to a post-20th century "modern" approach to harmony. Use of mathematical principals create form and structure to the piece, and while quite dissonant in some areas, an overall sense of aesthetic beauty was intended to be maintained.

CHAPTER THREE: THE WORK PROPER

(see attached)

CHAPTER FOUR: SUMMARY

In summary, I hope to bridge a gap of interdisciplinary knowledge from math to music, and to also better understand how we as humans perceive beauty and its various components. The application of this use of math is applicable to other fields as well. This is using math as a model to create a piece of music- math is used in much the same regard in many other fields as well: socially in statistical research and physically in such pursuits as actualizing architectural schemes. Iannis Xenakis, a Greek architect and composer, was perhaps the closest to actualizing this interdisciplinary relationship, using statistical models to write his compositions (Hoffman). His work, however, is relatively unknown to the vast majority of the population, and even amongst the populace that know of him, only a small portion truly appreciate his work. I wish for my composition to be heard as beautiful by the general populace, even with the semi-abandonment of Common Practice Era tonality and reliance instead on math as a tool for creation. When this happens I believe a true interdisciplinary work will have been born into this world.

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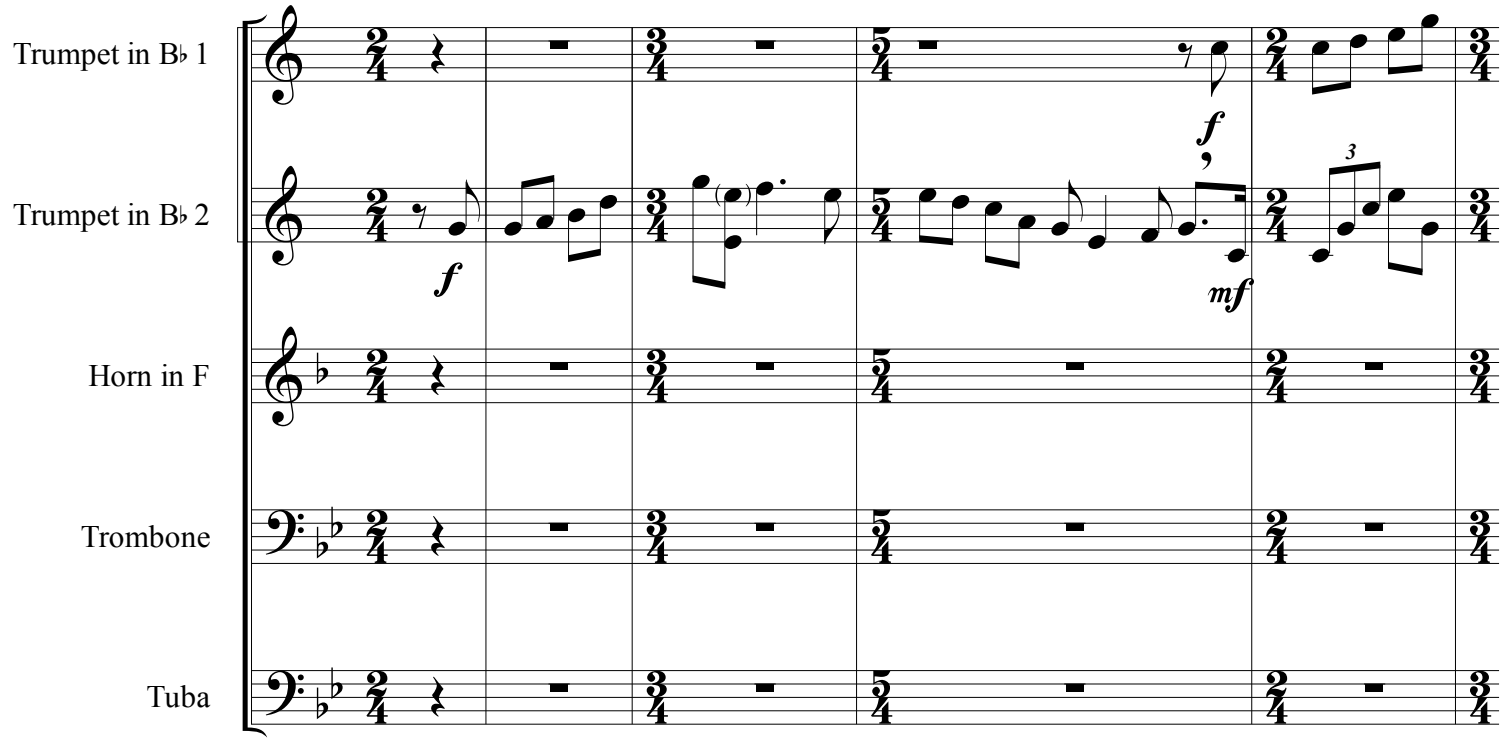
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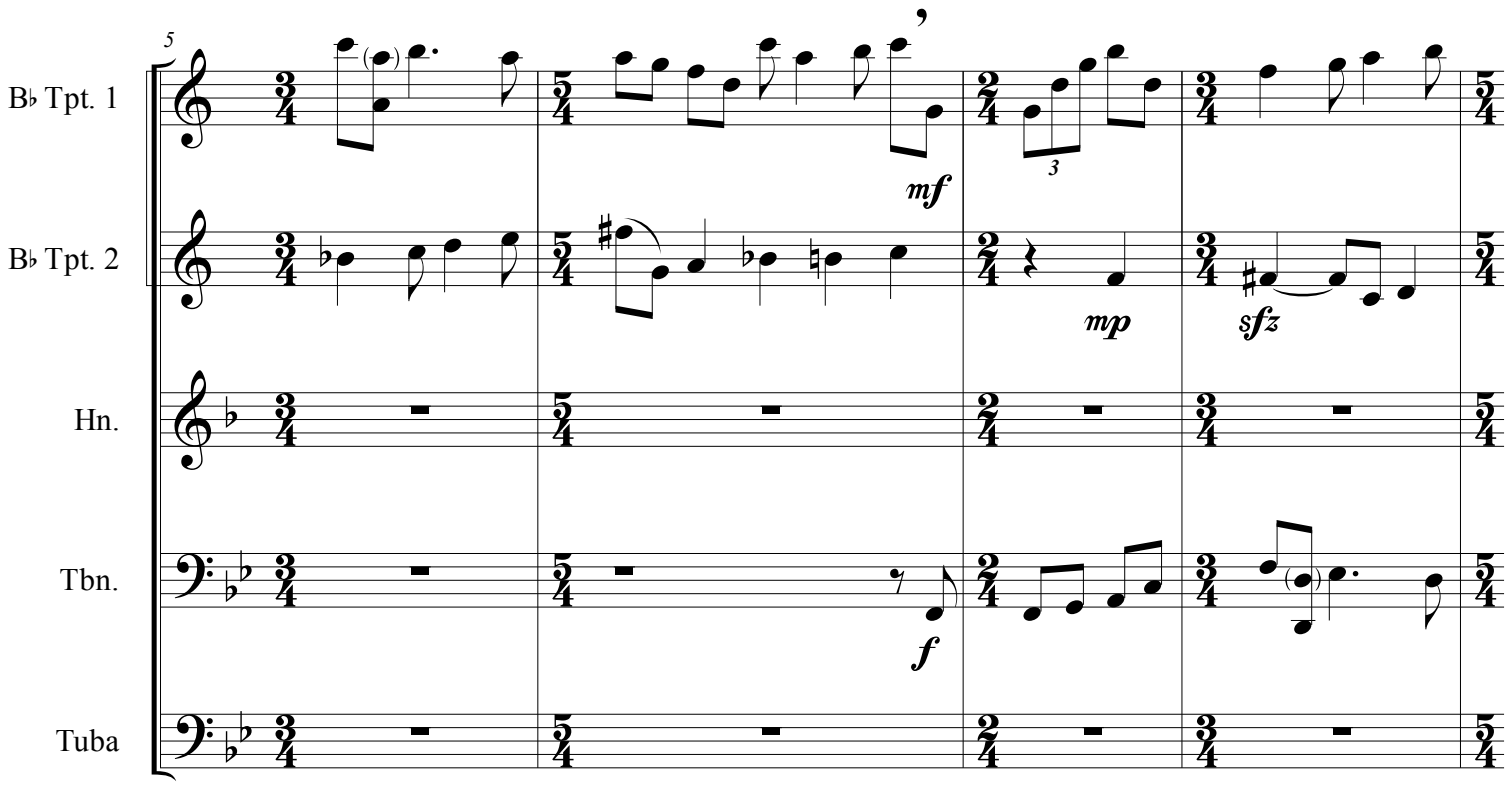
Fugue

for Brass Quintet

Stefan Greenfield-Casas
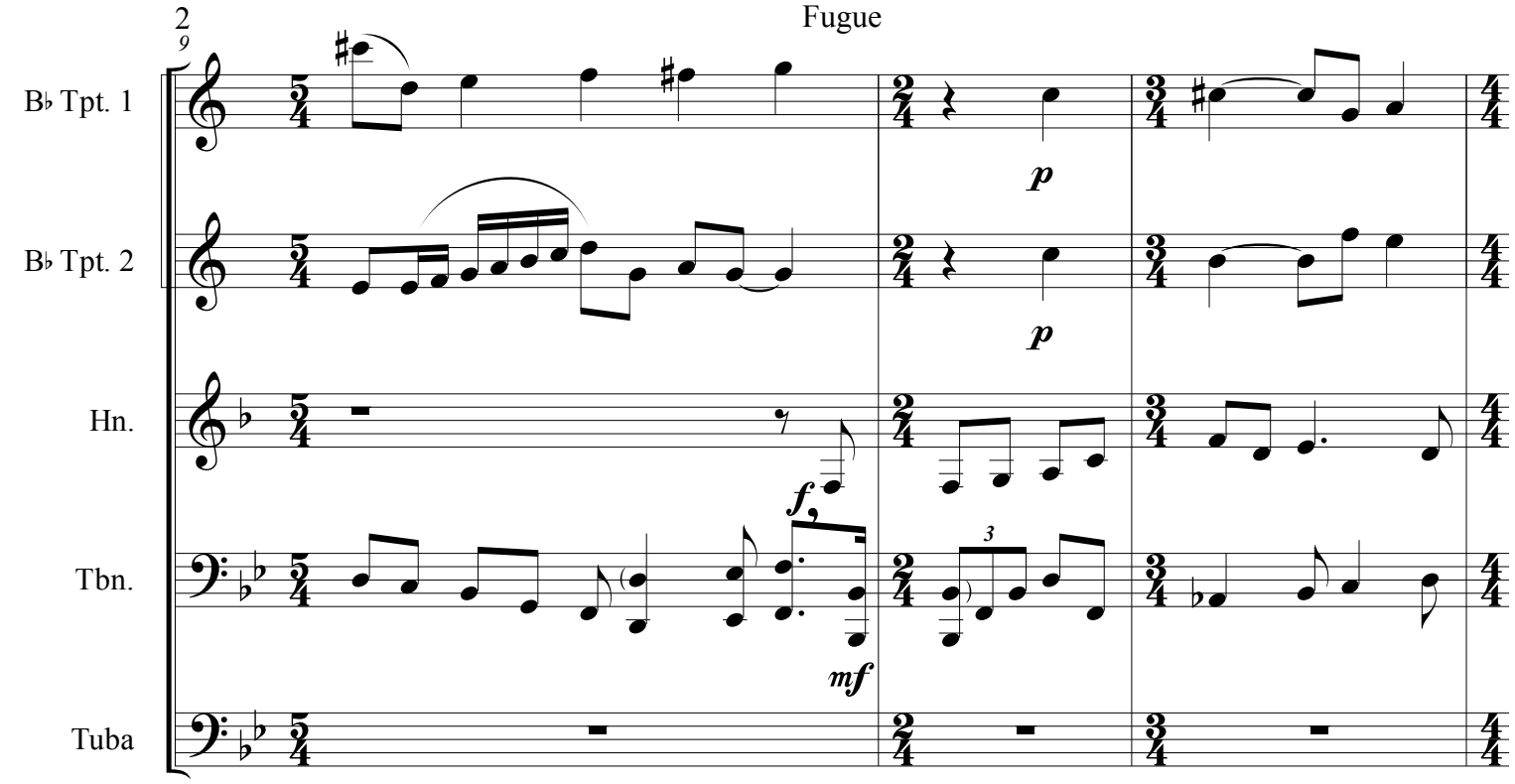
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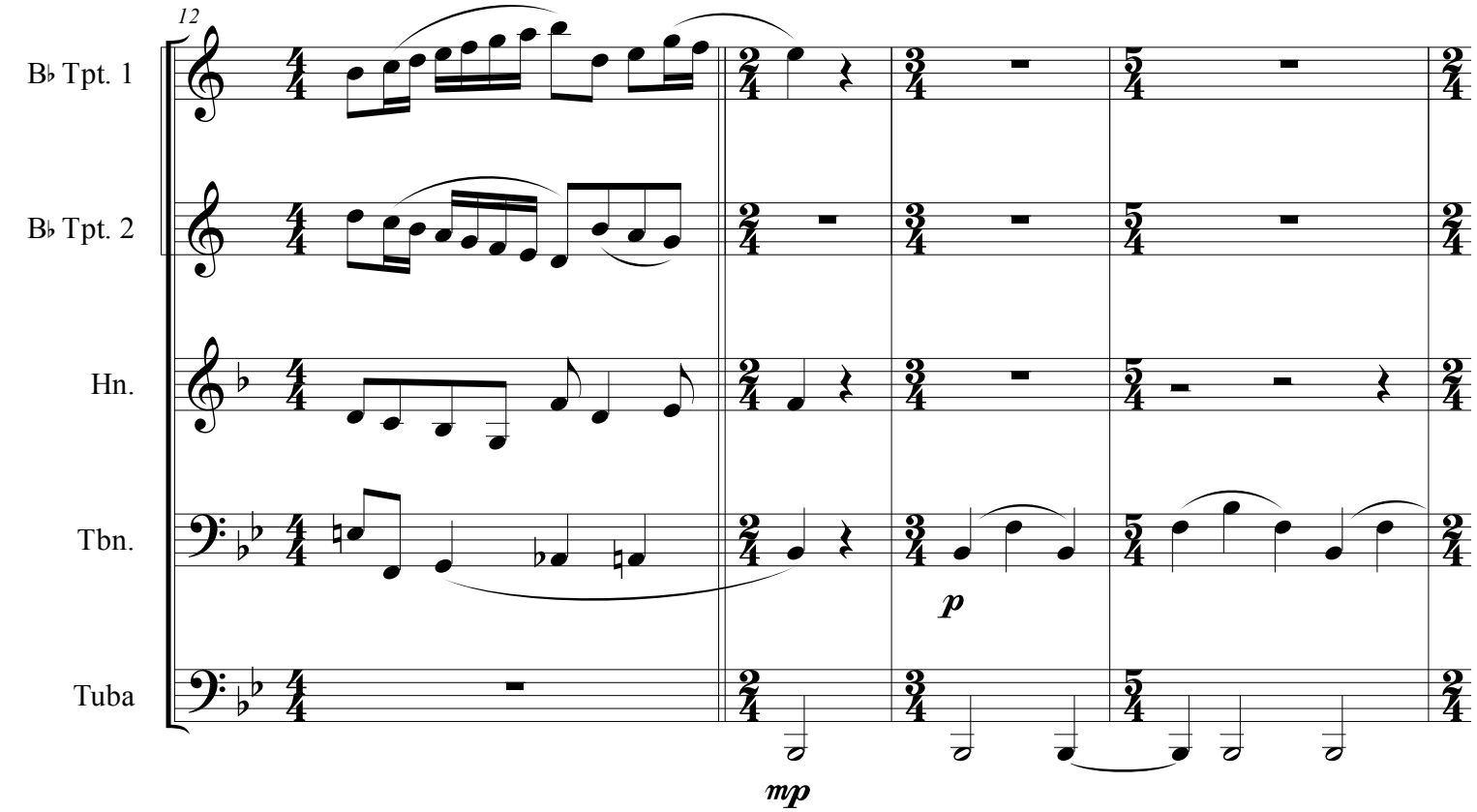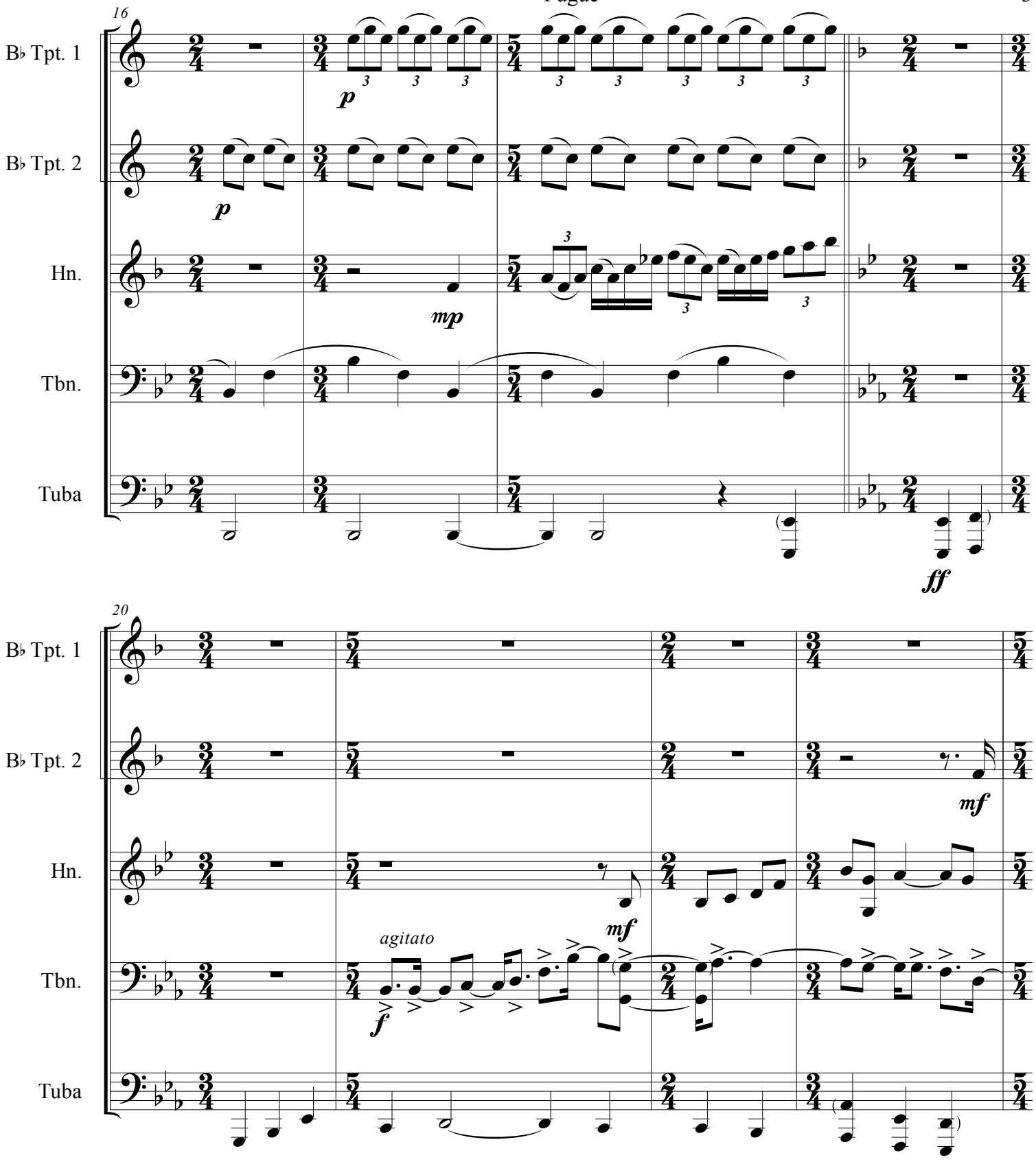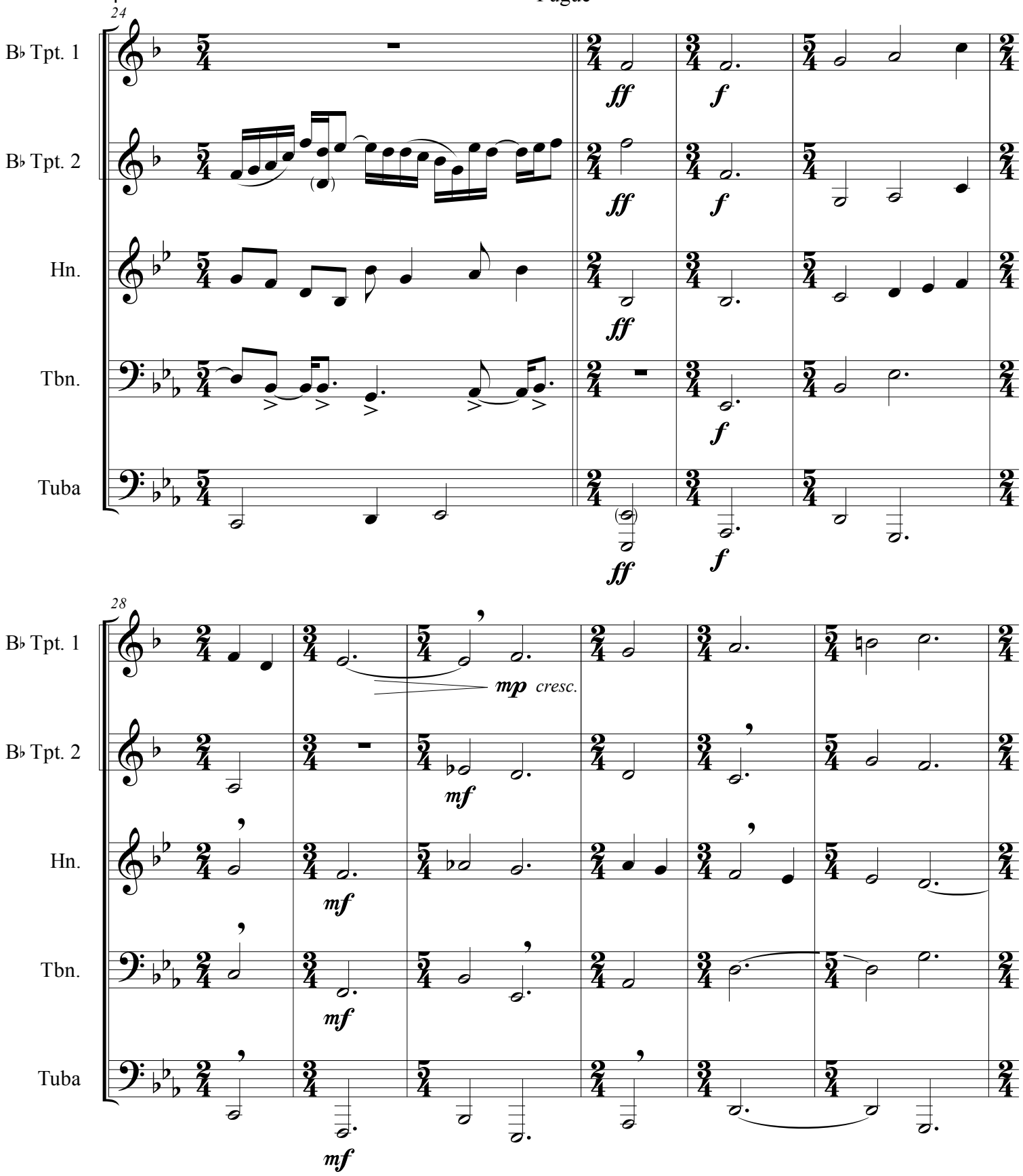




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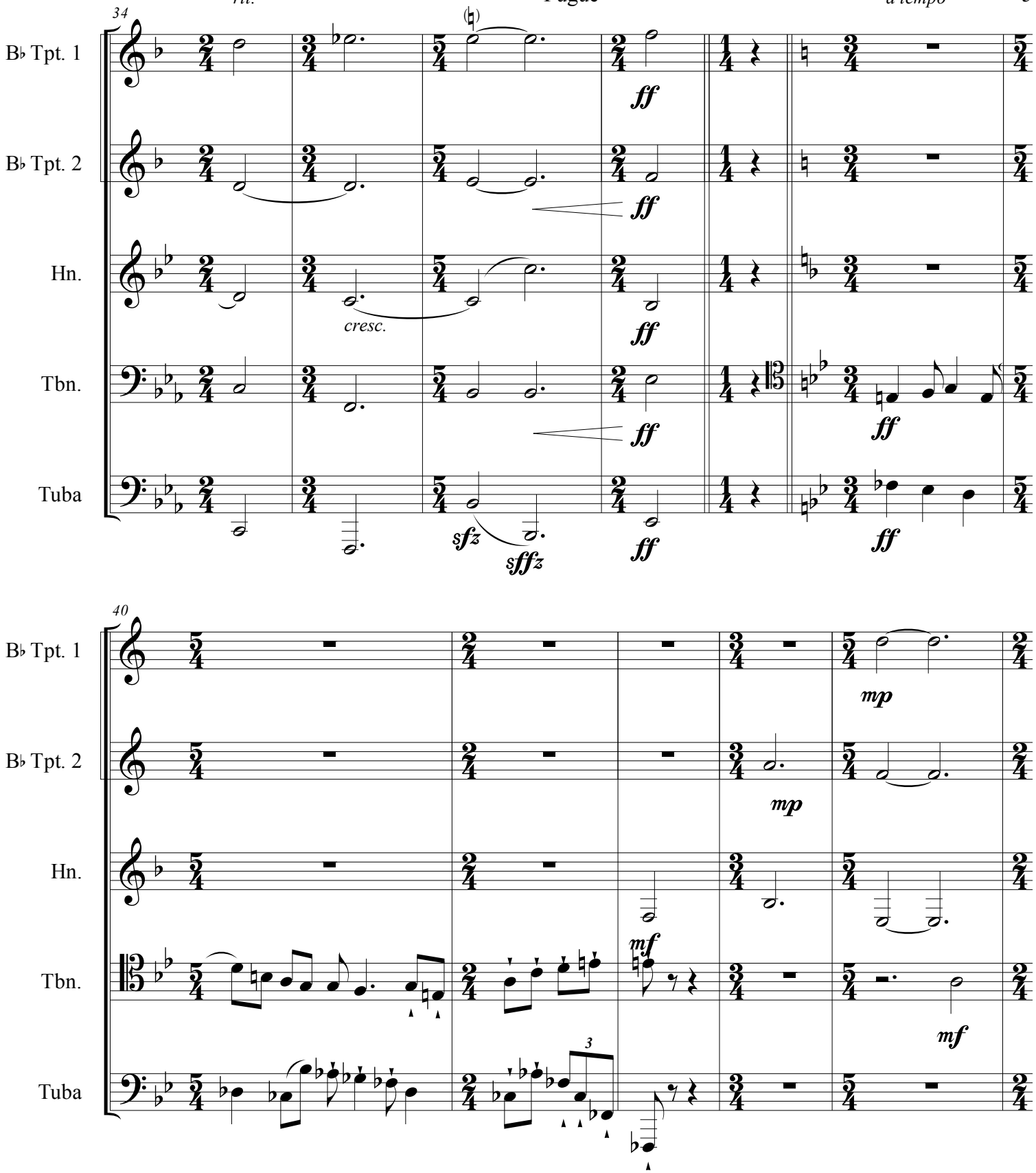
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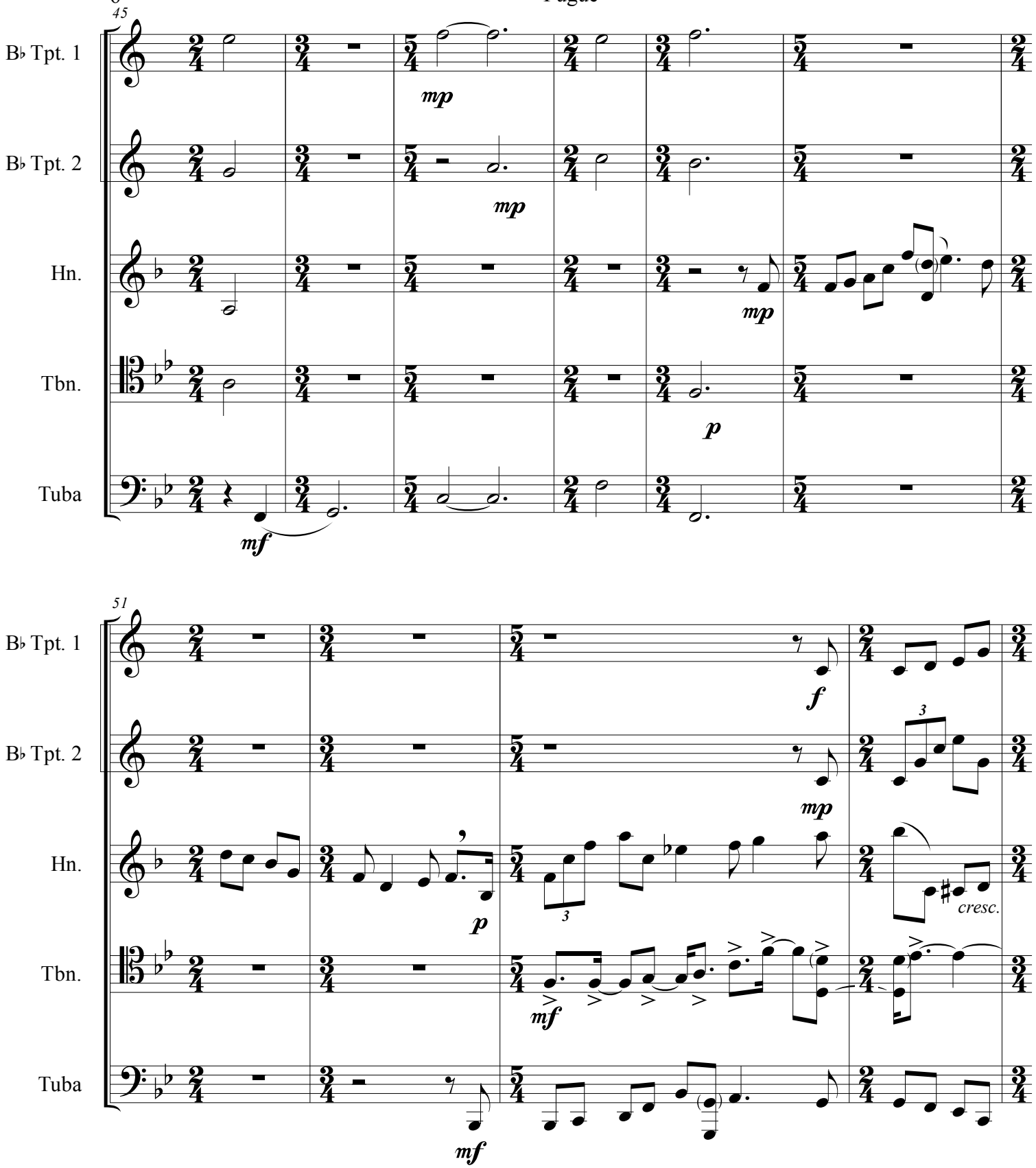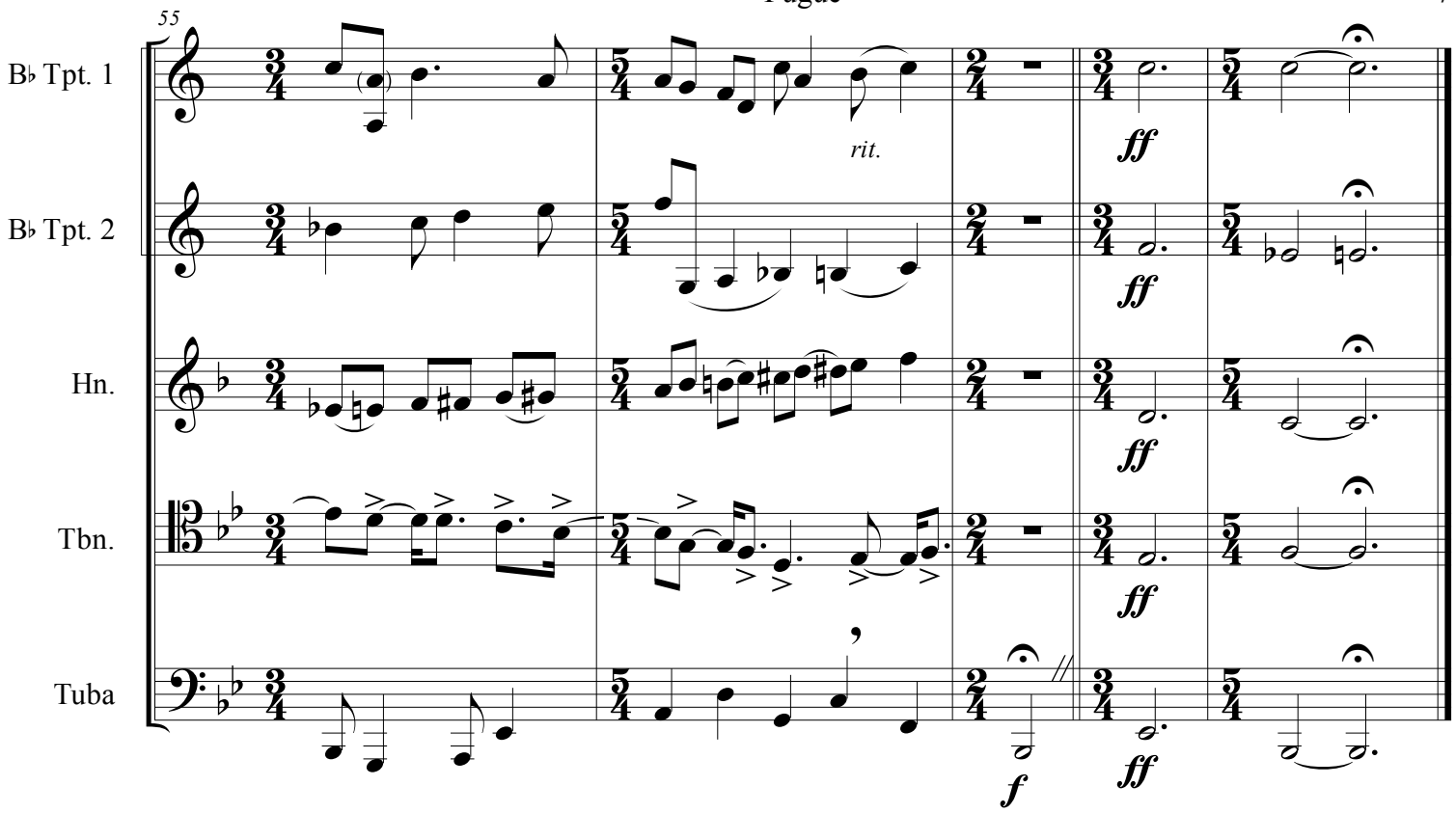





16
B♭ Tpt. 1
B♭ Tpt. 2
Hn.
Tbn.
Tuba
p
p
mp
ff
20
B♭ Tpt. 1
B♭ Tpt. 2
Hn.
Tbn.
Tuba
mf
mf
agitato
f


24
B♭ Tpt. 1
B♭ Tpt. 2
Hn.
Tbn.
Tuba
ff
f
28
mp cresc.
mf


rit.
a tempo
34
B♭ Tpt. 1
B♭ Tpt. 2
Hn.
Tbn.
Tuba
cresc.
ff
sfz
sffz
40
mp
mf
3


45
B♭ Tpt. 1
B♭ Tpt. 2
Hn.
Tbn.
Tuba
mp
mp
mp
p
mf
51
B♭ Tpt. 1
B♭ Tpt. 2
Hn.
Tbn.
Tuba
f
mp
p
mf
mf
3
3
cresc.


55
B♭ Tpt. 1
B♭ Tpt. 2
Hn.
Tbn.
Tuba
rit.
ff
f